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Q.1. Define countable set and prove that countable union of countable sets is countable.

Soln: Countable Set: →
A set which is either finite or denumerable is said to be 'countable'.
Let any set X can be mapped one-one onto the set $\{1, 2, \cdots n\}$, where $n \in N$, then X is called a countable set, we also say that X has n elements.
In this case, if $X = \{1, 2, \cdots n\}$ is established by function f and $1 \le K \le n$, then under f, the image of $K \in \{1, 2, \cdots n\}$ is denoted as f_K.
Hence, we write $X = \{f_1, f_2, \cdots f_n\}$

proof: Let $\{f_1, f_2, \cdots f_n, \cdots\}$ is a denumerable collection of sets where each f_n is denumerable.
Then, $f_n = \{a_{n1}, a_{n2}, a_{n3}, \cdots, a_{nn}, \cdots\}$ for each $n \in N$.

Thus, $f_1 = \{a_{11}, a_{12}, a_{13}, \cdots, a_{1n}, \cdots\}$
$f_2 = \{a_{21}, a_{22}, a_{23}, \cdots, a_{2n}, \cdots\}$
$f_3 = \{a_{31}, a_{32}, a_{33}, \cdots a_{3n}, \cdots\}$
$f_4 = \{a_{41}, a_{42}, a_{43}, \cdots a_{4n}, \cdots\}$
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Now, we can list all the elements of $f = \bigcup_{i=1}^{\infty} f_i$ in a sequence as follows.

We first list a_{11}, and then we list a_{21}, a_{12} and then a_{31}, a_{22} and a_{13} and thus proceed diagonally. In this process we leave out those elements of any diagonal which already occur on any previous diagonal.

Thus, we can arrange the elements of $f = \bigcup_{i=1}^{\infty} f_i$ in a sequence by naming the objects as we meet them in the above procedure as $b_1, b_2, b_3, \cdot$ (Hence, $b_1 = a_{11}$). Thus $f = \bigcup_{i=1}^{\infty} f_i = \{b_1, b_2, b_3, \dots\}$. Hence, f is denumerable and its cardinality is $\aleph_0$.

In this way countable union of countable sets is countable. proved

Q.2. Define a partial order relation on a set and illustrate the concept with two examples.

Soln. Partial order relation: → let X is any non-empty set and R is a relation in X then R is called a partial order relation in X when R is antisymmetric and transitive. If we write R as $\leq$, then

'$\leq$' is a partial order if

(I) For every $x, y \in X$, if $x \leq y$ and $y \leq x$ then $x = y$

II For every $x, y, z \in X$, if $x \leq y$ and $y \leq z$, then $x \leq$

The set $(X, \leq)$ is then called a partially ordered set.

Example I. For any set X, the inclusion relation $\subseteq$ is a partial order in $P(X)$.

Example II. In the set N of Natural Numbers the relation of divisibility '$|$' is a partial order, if $m|n$ we write $m \leq n$.